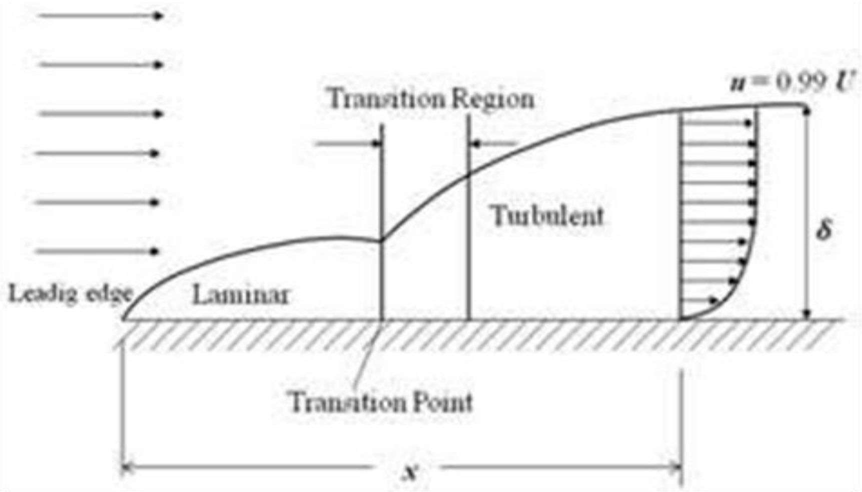
FLUID MECHANICS
(BTME 301-18)

Unit 6: Internal Flows

BOUNDARY LAYER CHARACTERISTICS

- Boundary layer is the regions close to the solid boundary where the effects of viscosity are experienced by the flow.
- In the regions outside the boundary layer, the effect of viscosity is negligible and the fluid is treated as inviscid.
- So, the boundary layer is a buffer region between the wall below and the inviscid free-stream above.
- This approach allows the complete solution of viscous fluid flows which would have been impossible through Navier-Stokes equation.

REPRESENTATION OF BOUNDARY LAYER ON A FLAT PLATE



- The qualitative picture of the boundary-layer growth over a flat plate is shown in Fig.
- A laminar boundary layer is initiated at the leading edge of the plate for a short distance and extends to downstream.
- The transition occurs over a region, after certain length in the downstream followed by fully turbulent boundary layers.
- For common calculation purposes, the transition is usually considered to occur at a distance where the Reynolds number is about 500,000. With air at standard conditions, moving at a velocity of 30m/s, the transition is expected to occur at a distance of about 250mm.

- A typical boundary layer flow is characterized by certain parameters as given below
  - Boundary thickness
  - Free stream flow (no viscosity)
  - Concepts of displacement thickness

BOUNDARY LAYER THICKNESS

- It is known that no-slip conditions have to be satisfied at the solid surface: the fluid must attain the zero velocity at the wall.
- Subsequently, above the wall, the effect of viscosity tends to reduce and the fluid within this layer will try to approach the free stream velocity.
- Thus, there is a velocity gradient that develops within the fluid layers inside the small regions near to solid surface.

- The *boundary layer thickness* is defined as the distance from the surface to a point where the velocity is reaches 99% of the free stream velocity.
- Thus, the velocity profile merges smoothly and asymptotically into the free stream as shown in Fig. 2.

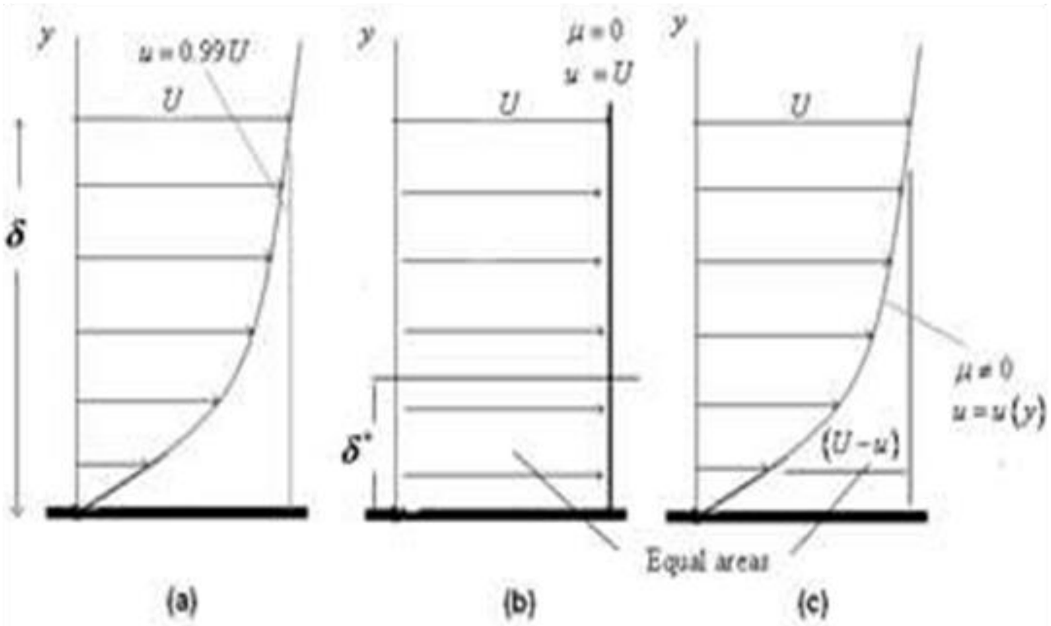


Fig. 5.7.3: (a) Boundary layer thickness; (b) Free stream flow (no viscosity); (c) Concepts of displacement thickness.

REYNOLD'S NUMBER

- Reynolds related the inertia to viscous forces and arrived at a dimension less parameter.

$$R_e \; or \; N_e = \frac{\text{inertia force}}{\text{viscous force}} = \frac{F_i}{F_v}$$

- According to Newton's 2nd law of motion, the inertia force F_i is given by

$$F_i = \text{mass} \times \text{acceleration}$$

$$= \rho \times \text{volume} \times \text{acceleration} \qquad (\rho = \text{mass density})$$

$$= \rho \times L^3 \times \frac{L}{T^2} = \rho L^2 V^2 \quad \text{---- (1)} \qquad (L = \text{Linear dimension})$$

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- Similarly viscous force F_V is given by Newton's 2^{nd} law of velocity as

$$F_V = \tau \times \text{area} \qquad \tau = \text{shear stress}$$

$$= \mu \frac{dv}{dy} \times L^2 = \mu VL \quad \text{----------------} \quad (2)$$

V = Average Velocity of flow

μ = Viscosity of fluid

$$R_e \text{ or } N_R = \frac{\rho L^2 V^2}{\mu VL} = \frac{\rho VL}{\mu}$$

- In case of pipes L = D

- In case of flow through pipes

$$R_e = \frac{\rho D V}{\mu} \text{ or } \frac{VD}{\nu} \nu$$

Where μ/ρ = kinematic viscosity of the flowing liquid ν

- The Reynolds number is a very useful parameter in predicting whether the flow is laminar or turbulent.

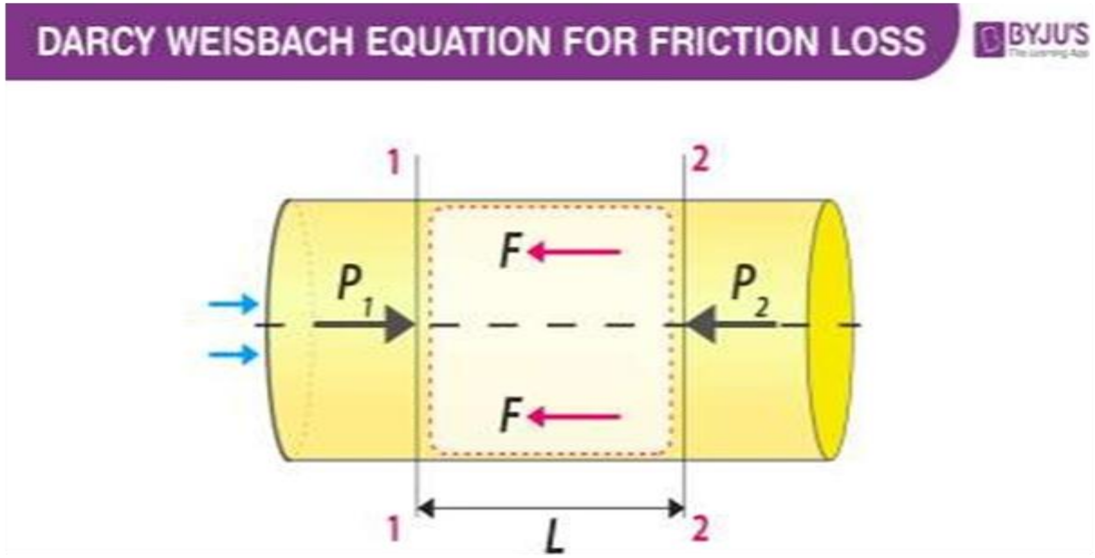
➢ $R_e < 2000$ viscous / laminar flow

➢ $R_e \rightarrow 2000$ to 4000 transient flow

➢ $R_e > 4000$ Turbulent flow

FRICTIONAL LOSS IN PIPE FLOW – DARCY WEISBACK EQUATION

- When a liquid is flowing through a pipe, the velocity of the liquid layer adjacent to the pipe wall is zero.
- The velocity of liquid goes on increasing from the wall and thus velocity gradient and hence shear stresses are produced in the whole liquid due to viscosity.
- This viscous action causes loss of energy, which is known as frictional loss.



DERIVATION

- Consider a uniform horizontal pipe having steady flow. Let 1-1, 2-2 are two sections of pipe.

Let P_1 = Pressure intensity at section 1-1

V_1 = Velocity of flow at section 1-1

L = Length of pipe between section 1-1 and 2-2

d = Diameter of pipe

f' = Fractional resistance for unit wetted area per a unit velocity

h_f = Loss of head due to friction

- And P_2,V_2 = are values of pressure intensity and velocity at section 2-2

- Applying Bernoulli's equation between sections 1-1 and 2-2

Total head at 1-1 = total head at 2-2 + loss of head due to friction between 1-1 and 2-2

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} + Z_2$$

$Z_1 = Z_2$ as pipe is horizontal

$V_1 = V_2$ as dia. of pipe is same at 1-1 and 2-2

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f \qquad \text{Or}$$

$$h_f = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} \qquad \text{———————} (1)$$

- But h_f is head is lost due to friction and hence the intensity of pressure will be reduced in the direction flow by frictional resistance.

- Now, Frictional Resistance = Frictional resistance per unit wetted area per unit velocity × Wetted Area × (velocity)^2

$F_1 = f' \times \pi d L \times V^2$ [∵Wetted area $= \pi d \times L$, Velocity = $V = V_1 = V_2$]

$F_1 = f' \times pLV^2$ ——————————(2) [∵ πd = perimeter = p]

The forces acting on the fluid between section 1-1 and 2-2 are

Pressure force at section 1-1 = $P_1 \times A$ (where A = area of pipe)

Pressure force at section 2-2 = $P_2 \times A$

Frictional force = F_1

- Resolving all forces in the horizontal direction, we have

$P_1A - P_2A - F_1 = 0$

$(P_1 - P_2)A = F_1 = f' \times p \times L \times V^2$ from equation - (2)

$P_1 - P_2 = \frac{f^F \times p \times L \times V^2}{A}$ But from equation (1) $P_1 - P_2 = \rho g h_f$

- Equating the value of $P_1 - P_2$, we get

$$\rho g h_f = \frac{f^F \times p \times L \times V^2}{A}$$

$$h_f = \frac{f^F}{\rho g} \times \frac{P}{A} \times L \times V^2 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(3)}$$

- In the equation (3) $\frac{P}{A} = \frac{\text{Wetted Permiter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{\pi}{d}$

$$h_f = \frac{f^F}{\rho g} \times \frac{4}{d} \times L \times V^2 = \frac{f^F}{\rho g} \times \frac{4LV^2}{d}$$

Putting $\frac{f^F}{\rho} = \frac{f}{2}$ Where f is known as co-efficient of friction.

- Equation (4) becomes as $h_f = \frac{4f}{2g} \times \frac{LV^2}{d}$

$$h_f = \frac{4fLV^2}{2gd}$$

- This Equation is known as Darcy - Weisbach equation, commonly used for finding loss of head due to friction in pipes
- Then f is known as a friction factor or co-efficient of friction which is a dimensionless quantity. f is not a constant but, its value depends upon the roughness condition of pipe surface and the Reynolds number of the flow.

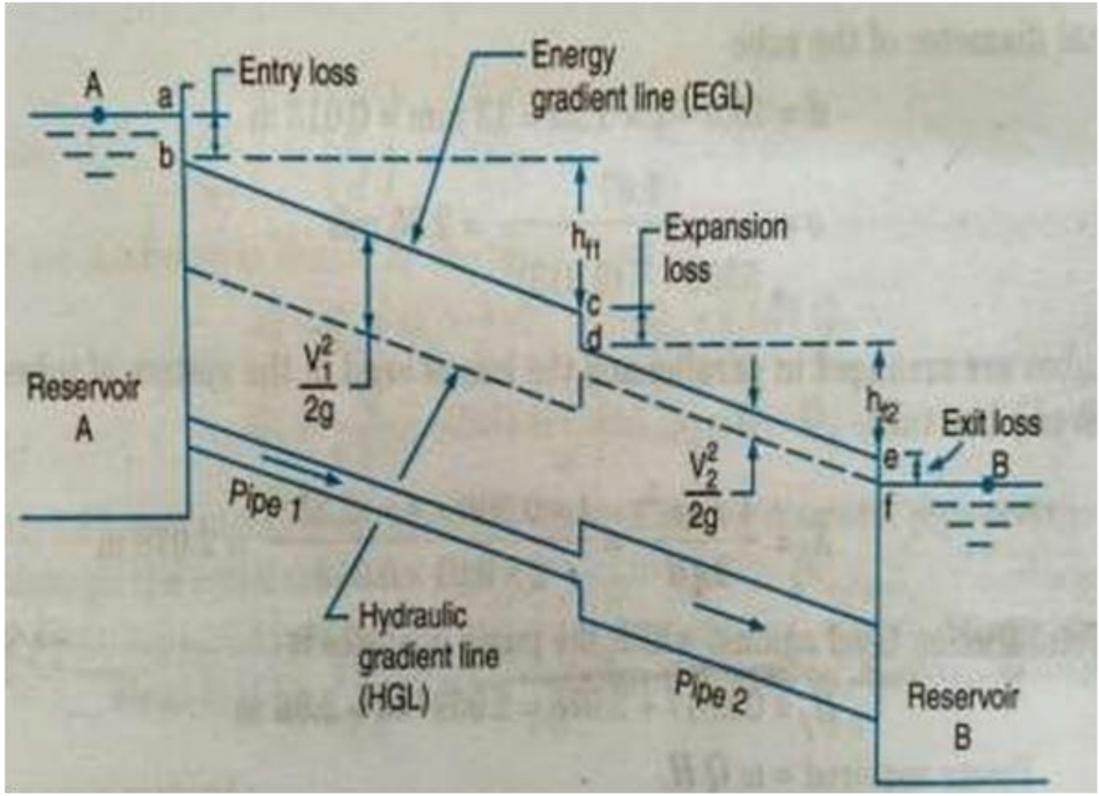
MINOR LOSSES IN PIPES

- The loss of energy due to friction is classified as a major loss, because in case of long pipe lines it is much more than the loss of energy incurred by other causes.
- The minor losses of energy are caused on account of the change in the velocity of flowing fluids (either in magnitude or direction).
- In case of long pipes these loses are quite small as compared with the loss of energy due to friction and hence these are termed as ''minor losses ''
- Which may even be neglected without serious error, however in short pipes these losses may sometimes outweigh the friction loss.
- Some of the losses of energy which may be caused due to the change of velocity are:

MINOR LOSSES IN PIPES

- Loss of energy due to sudden enlargement, $h_e = \frac{(V_1 - V_2)^2}{2g}$
- Loss of energy due to sudden contraction, $h_c = 0.5 \frac{V_2^2}{2g}$
- Loss of energy at the entrance to a pipe, $h_i = 0.5 \frac{V^2}{2g}$
- Loss of energy at the exit from a pipe, $h_o = \frac{V^2}{2g}$
- Loss of energy due to gradual contraction or enlargement, $h_l = \frac{k(V_1 - V_2)^2}{2g}$
- Loss of energy in the bends, $h_b = \frac{kV^2}{2g}$
- Loss of energy in various pipe fittings, $h_l = \frac{kV^2}{2g}$

- Consider a long pipe line carrying liquid from a reservoir A to reservoir B.
- At several points along the pipeline let piezometers be installed.



- On account of loss of energy due to friction, the pressure head will decrease gradually from section to section of pipe in the direction of flow.
- If the pressure heads at the different sections of the pipe are plotted to scale as vertical ordinates above the axis of the pipe and all these points are joined by a straight line, a sloping line is obtained, which is known as Hydraulic Gradient Line (H.G.L).
- Since at any section of pipe the vertical distance between the pipe axis and Hydraulic gradient line is equal to the pressure head at that section, it is also known as pressure line.
- Moreover if Z is the height of the pipe axis at any section above an arbitrary datum, then the vertical height of the Hydraulic gradient line above the datum at that section of pipe represents the piezometric head equal to (p/w + z)..

- Sometimes the Hydraulic gradient line is also known as piezometric head line.
- At the entrance section of the pipe for some distance the Hydraulic gradient line is not very well defined.
- This is because as liquid from the reservoir enters the pipe, a sudden drop in pressure head takes place in this portion of pipe.
- Further the exit section of pipe being submerged, the pressure head at this section is equal to the height of the liquid surface in the reservoir B and hence the hydraulic gradient line at the exit section of pipe will meet the liquid surface in the reservoir B.